Towards flying through modular forms

David Lowry-Duda and Adam Sakareassen

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ICERM and Brown University

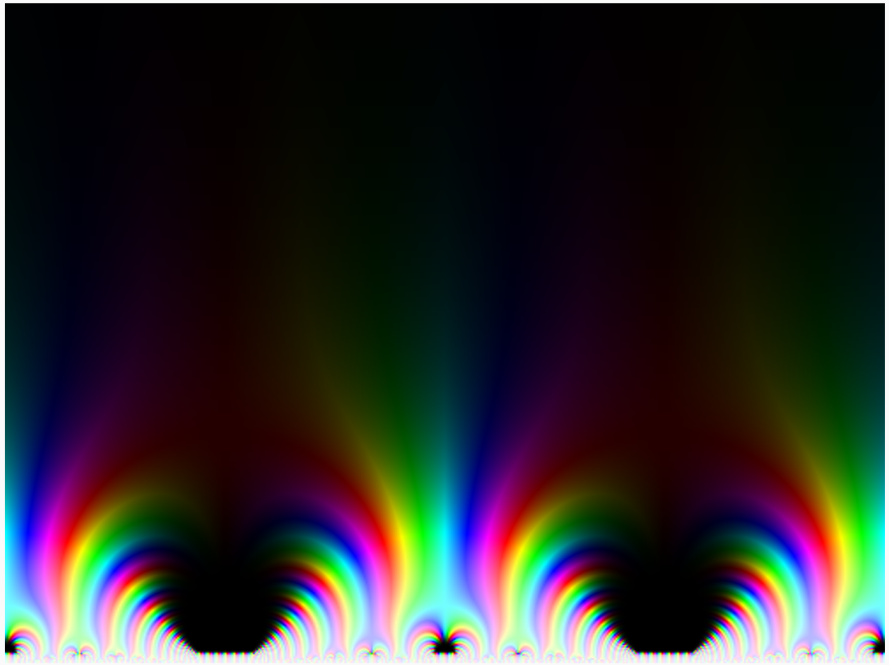
I research number theory. Frequently, I use "highly symmetric functions" called *modular forms*, whose definition is unilluminating:

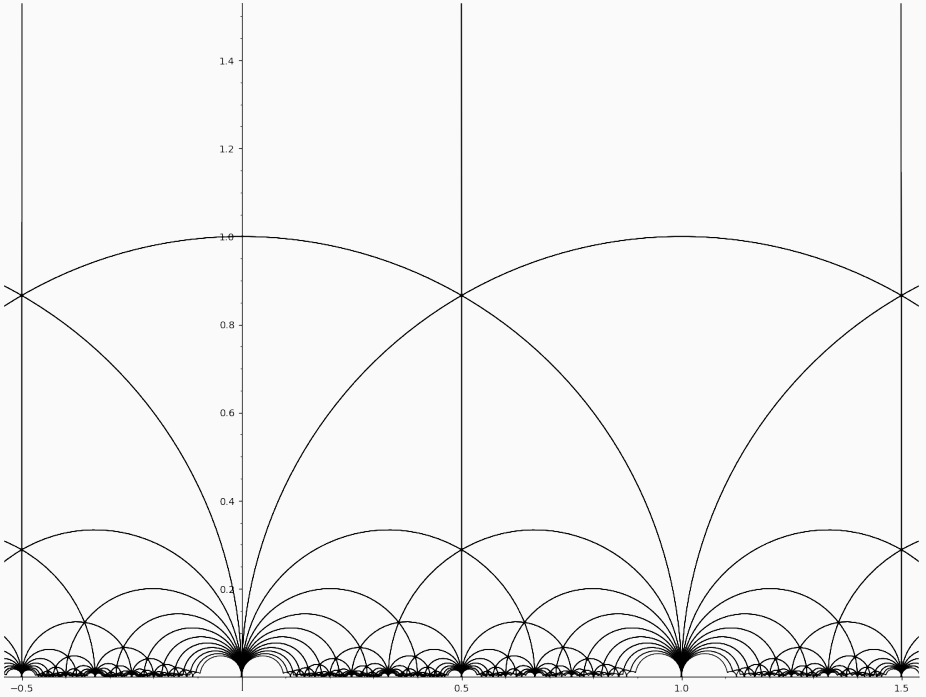
A modular form f is a complex-valued function from the half-plane to $\mathbb{C}$ that satisfies an infinite number of functional equations of the shape

$$f\left(\frac{az+b}{cz+d}\right) = (cz+d)^k f(z)$$

for various quadruples $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ and some fixed integer k.

I've written several papers exploiting modular forms, or describing how to compute them, or how to make sense of their properties. Cecently (inspired in large part by the 2019 ICERM program *Visualizing Mathematics*), I began to study what modular forms look like?

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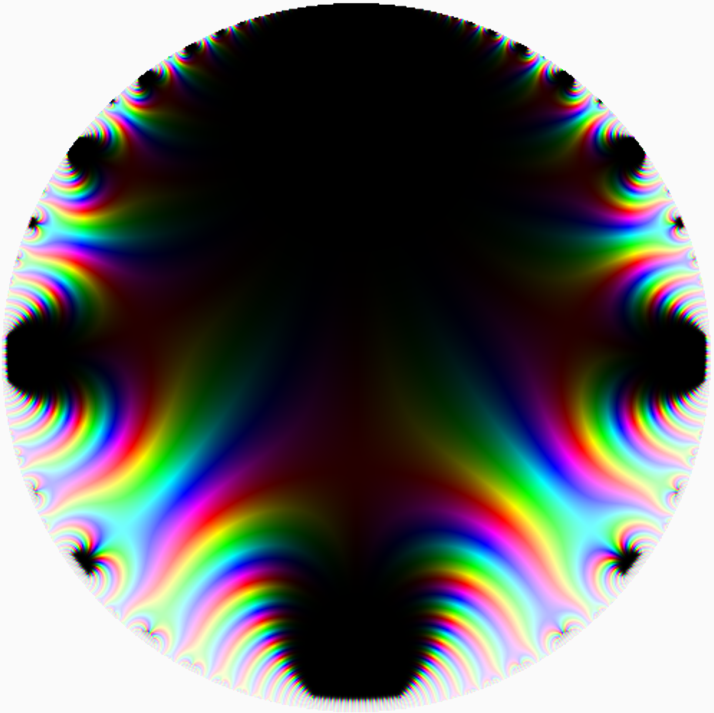
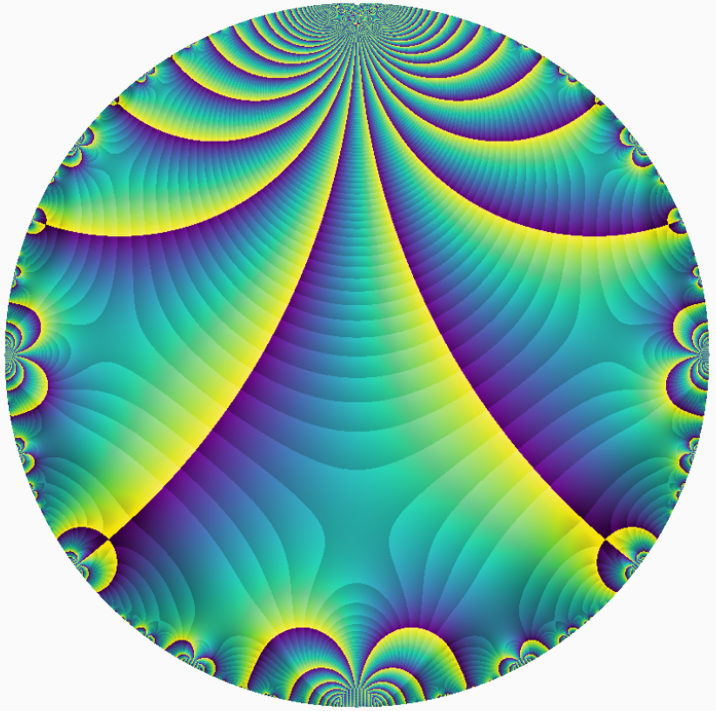
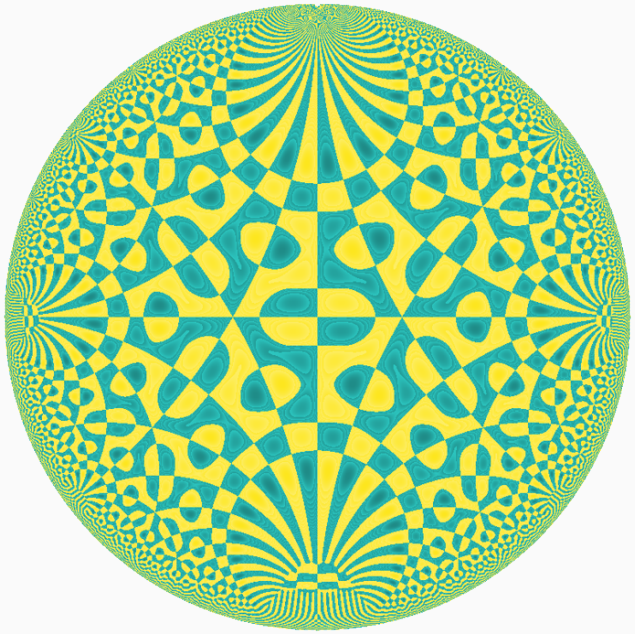
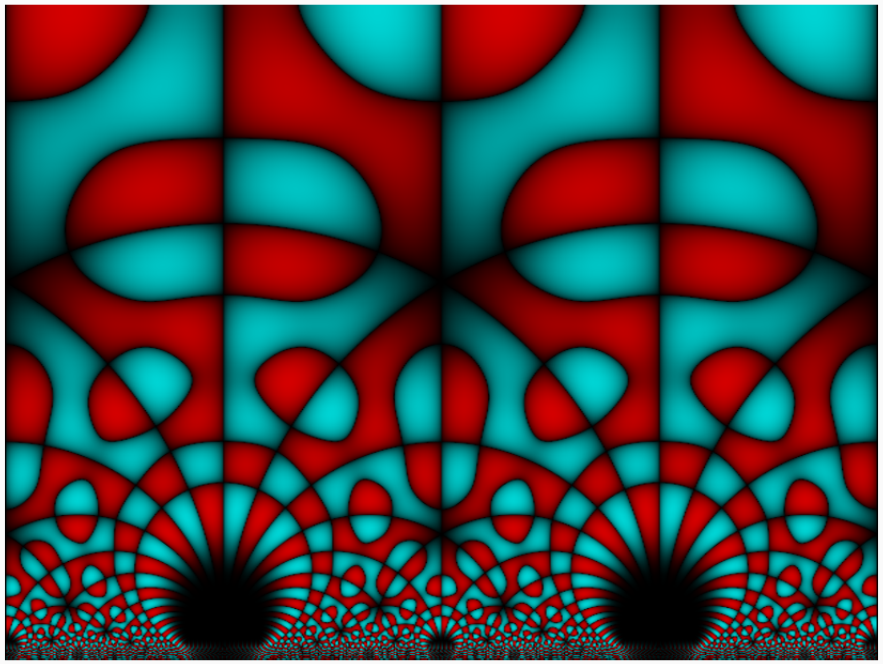
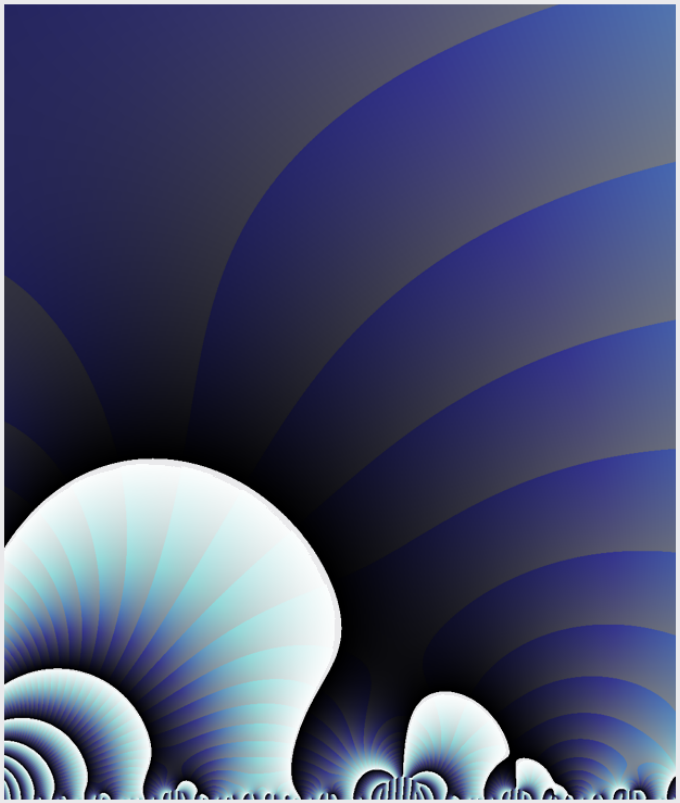
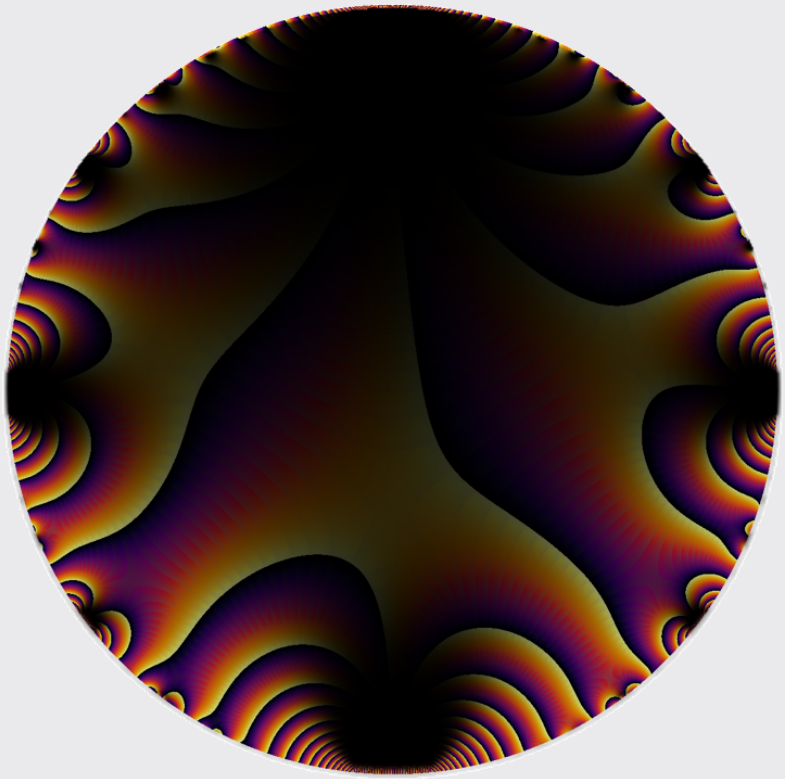


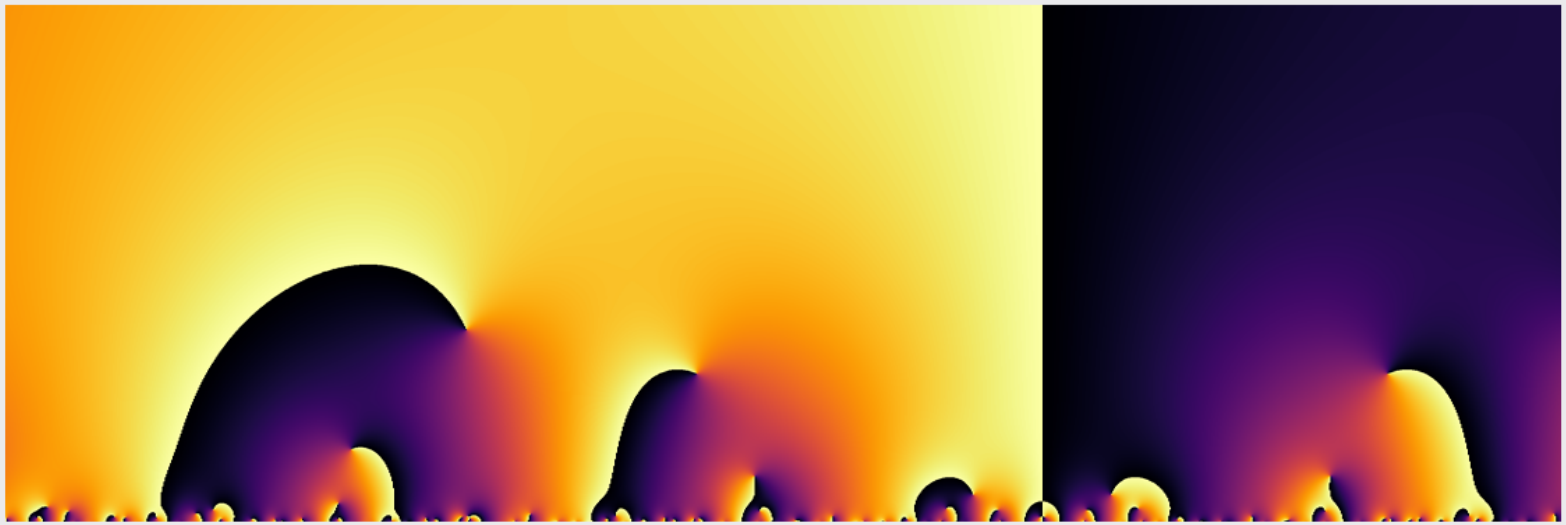




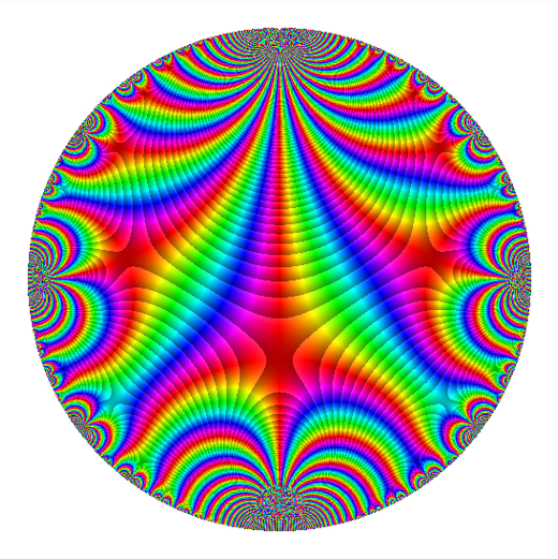


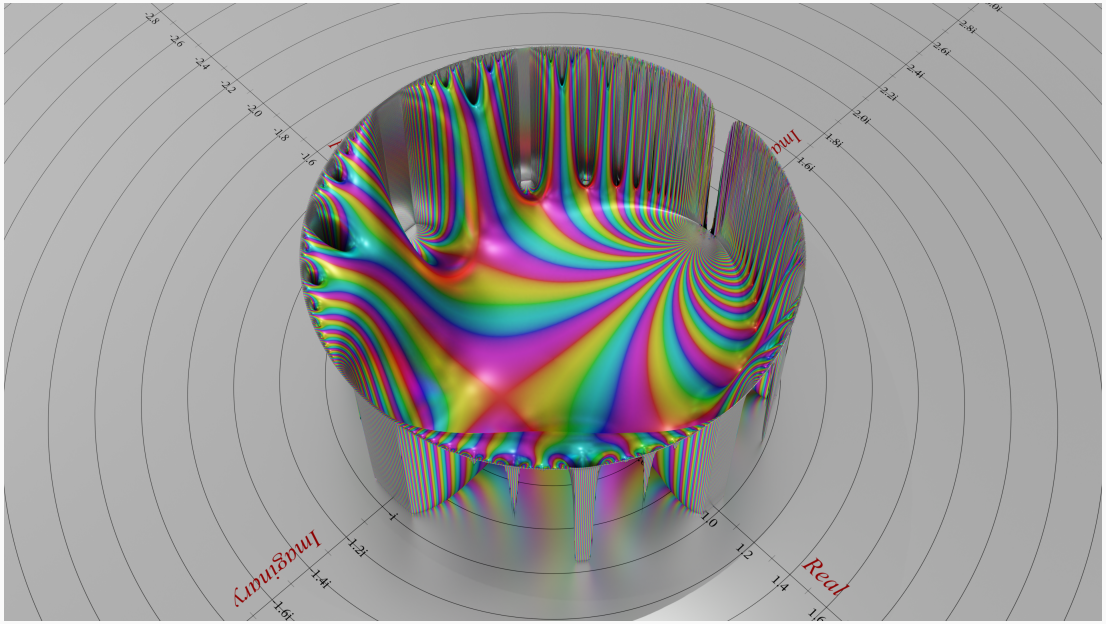


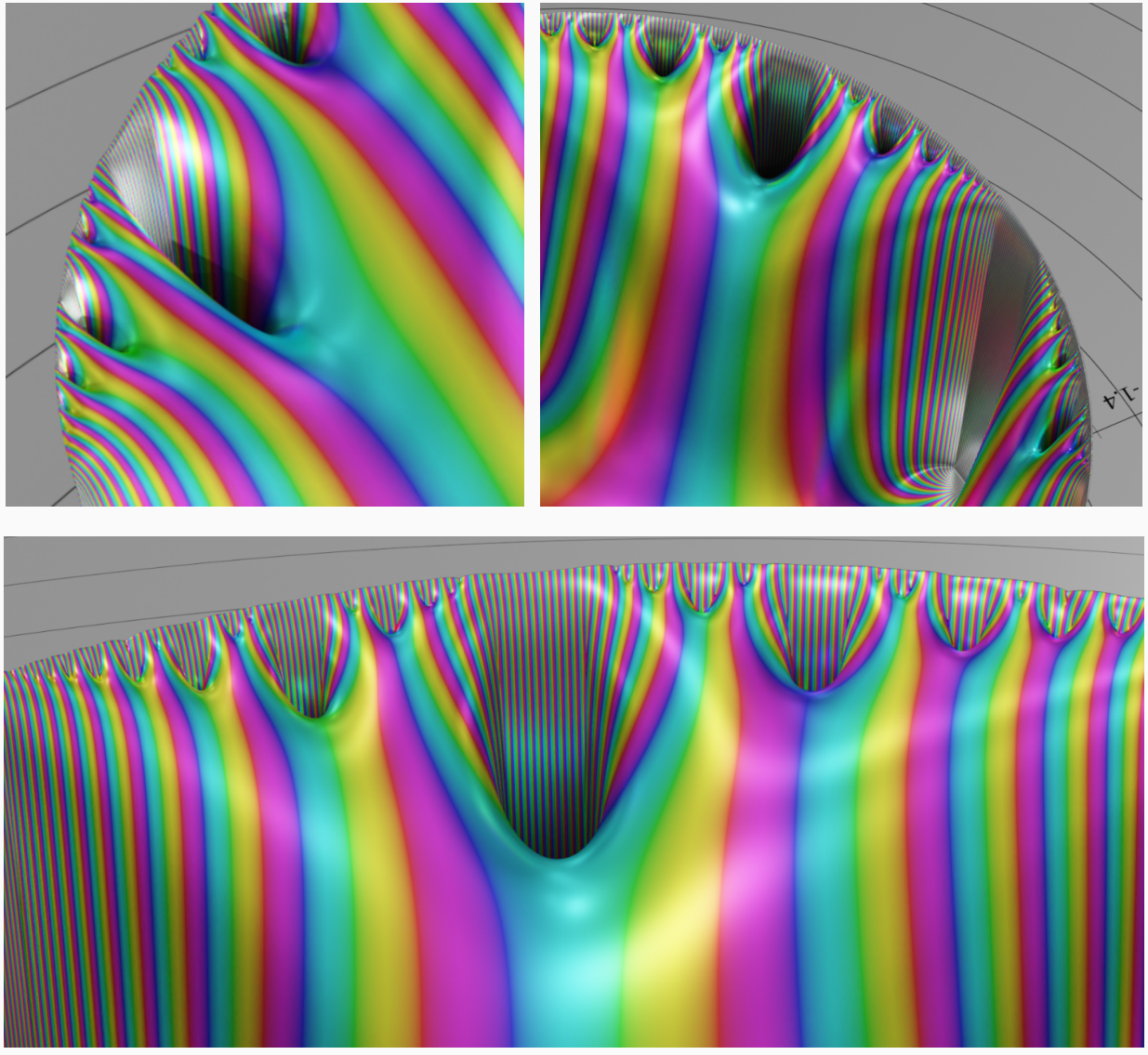




With Adam Sakareassen (who has taught me everything I know about quality 3d rendering), we began to explore modular forms in 3d.



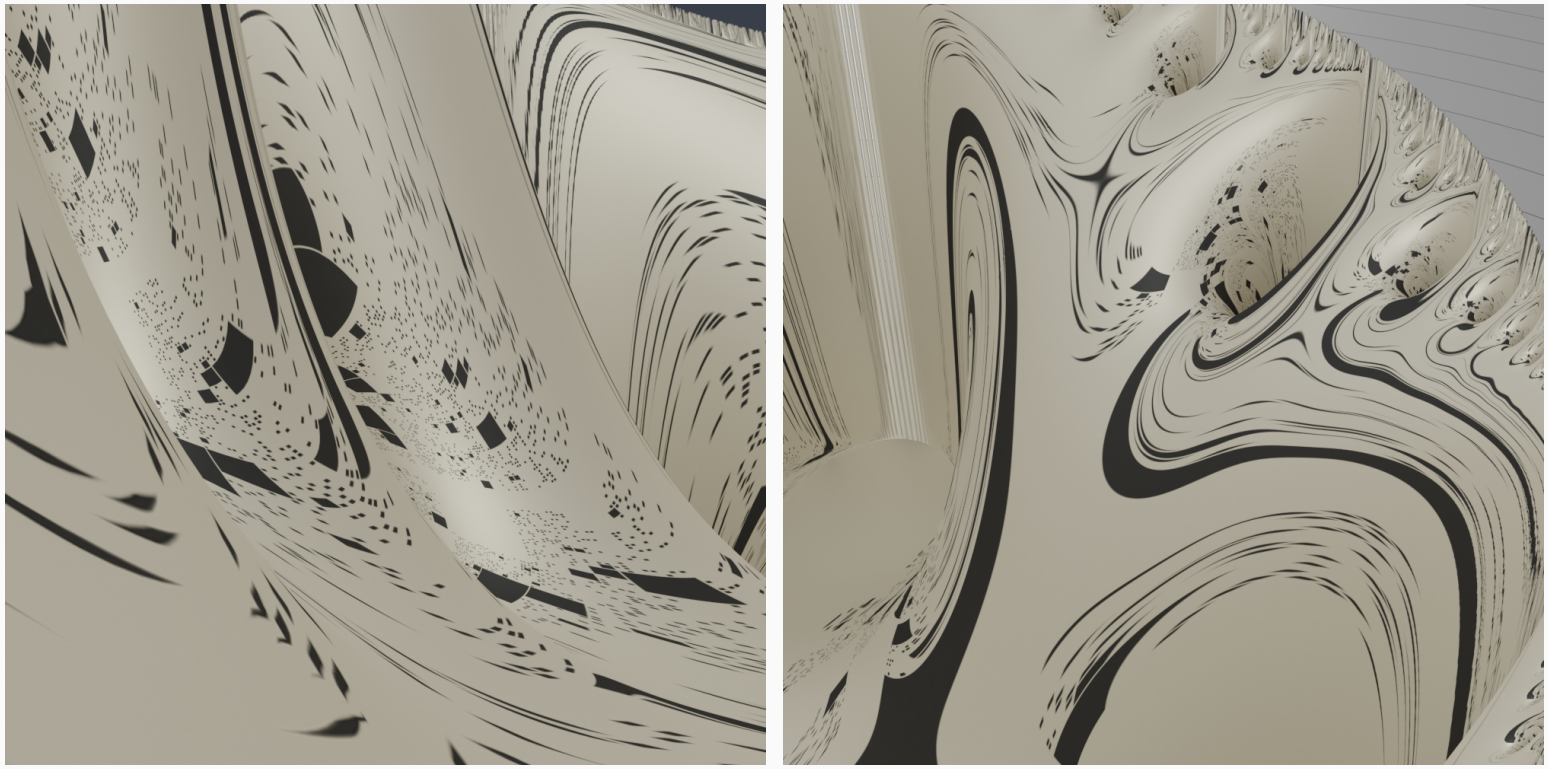


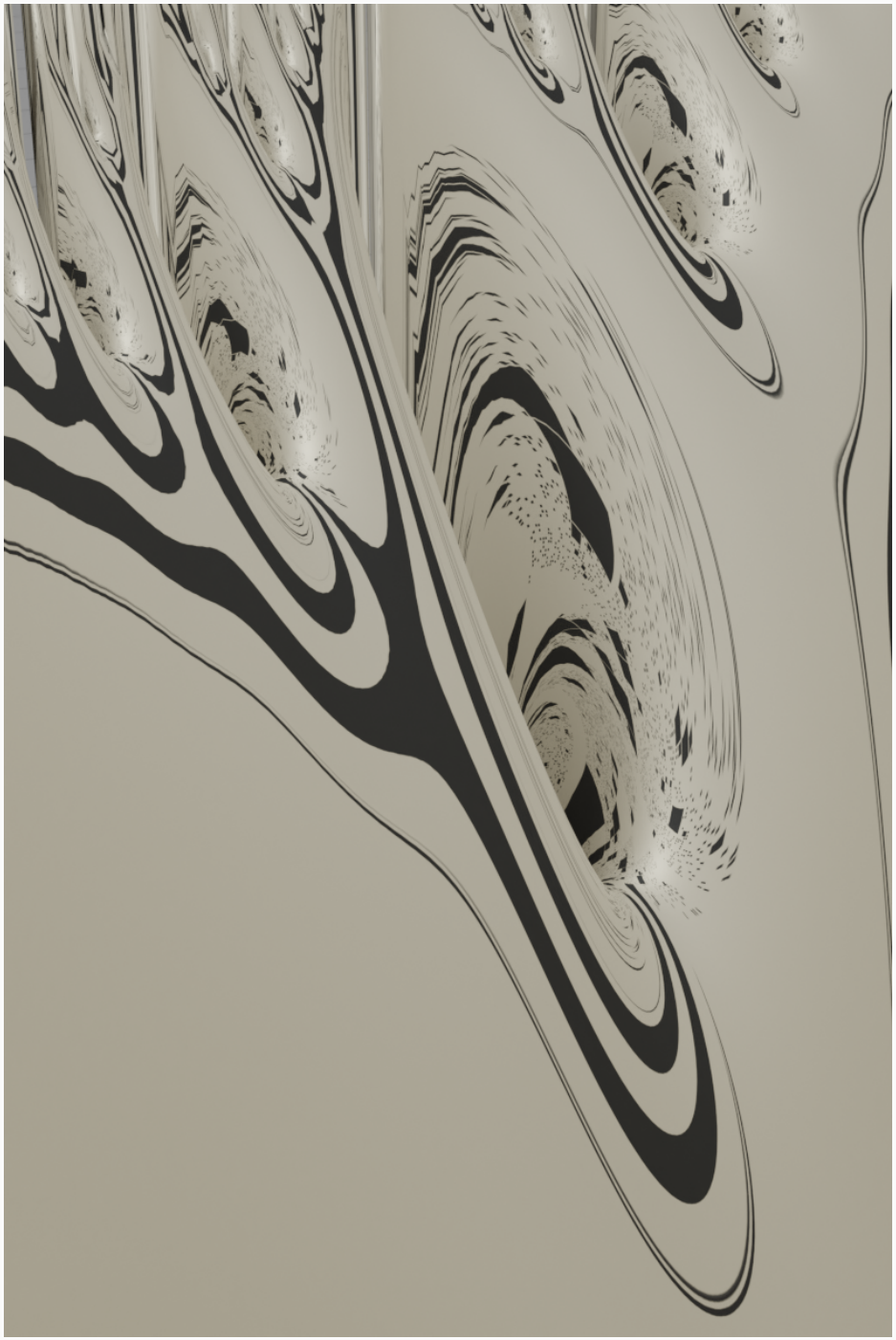


We've begun to make not just 3d images, but movies and gifs.

Our first is at `https://www.youtube.com/watch?v=s6sdEbGNdic`

We're taking this in different directions. As before, we began with the *informational* and we're moving towards both *educational* and *beautiful*.





And also, a bit of the ridiculous. (This is my wife and I visiting Iceland, standard on the left and *symmetrized* and used as a modular form wallpaper on the right).

Thank you very much!

More visualizations and details about modular forms appear on

davidlowryduda.com

visual.davidlowryduda.com