

INTRODUCTION TO NUMBER THEORY
Spring 2016

Homework # 6

Last Updated: March 11, 2016

Due Date: Thursday March 24th.

I recommend you read Chapters 9, 10, and 11.

FRINT Chapter 11:

- (1) 11.6
- (2) 11.9
- (3) Find all solutions to

$$x^2 \equiv 1 \pmod{105}$$

by finding solutions to the three congruences

$$x^2 \equiv 1 \pmod{3}$$

$$x^2 \equiv 1 \pmod{5}$$

$$x^2 \equiv 1 \pmod{7}$$

and using the Chinese Remainder Theorem.

FRINT Chapter 12:

- (4) 12.1
- (5) 12.3 (parts a and b)

FRINT Chapter 13:

- (6) 13.1
- (7) 13.3. Conclude that there are arbitrarily large differences between consecutive primes.
- (8) 13.5
- (9) (In this exercise, you produce another proof of the infinitude of primes). Show that the integer $Q_n = n! + 1$ has a prime factor larger than n . Deduce from this that there are infinitely many primes.

FRINT Chapter 14:

- (10) 14.3

FRINT Chapter 15:

- (11) 15.1
- (12) 15.2
- (13) 15.6