Introduction to Number Theory

Spring 2016

Homework # 4
Last Updated: February 16, 2016
Due Date: Thursday February 25th

I recommend you read Chapters 1,2,3,5,6,7,8, and 9.

FRINT Chapter 8:

(1) 8.1
(2) 8.2
(3) 8.3
(4) 8.6 (You do not need to write down the solutions, but do write down how you know how many there are).

FRINT Chapter 9:

(5) 9.1

Additional UNGRADED Problems:

(6) We have been learning about the relationship between equations of the form $ax + by = c$ and congruence equations of the form $ax \equiv c \pmod{b}$. Let's try to make sure we understand how these are related.
   (a) Consider the Diophantine equation $8x + 14y = 6$. Without finding them, are there solutions?
   (b) Find a solution to $8x + 14y = 6$.
   (c) Find all solutions to $8x + 14y = 6$. (Two solutions are $(-1, 1)$ and $(6, -3)$. Does your expression include both?).
   (d) Consider the linear congruence $8x \equiv 6 \pmod{14}$. Without finding them, are there solutions?
   (e) Find a solution to $8x \equiv 6 \pmod{14}$.
   (f) Find all incongruent solutions to $8x \equiv 6 \pmod{14}$.
(7) Suppose that $\gcd(a, b) = 1$, and suppose that $a \mid bc$. Without using the Fundamental Theorem of Arithmetic (also called Unique Factorization), show that $a \mid c$.
(8) Find all solutions to $x^2 \equiv 1 \pmod{8}$.
(9) We found all primitive Pythagorean triples in two different ways. Do you remember them?
(10) We later found infinitely many square-triangular numbers. Do you remember how to set that problem up, and what the key ideas were to solve it? (What does this have to do with Pythagorean triples, or perhaps with our methods of finding Pythagorean triples?)