Math 0100

Week 10 Quiz - Version A

You have ten minutes to answer the following question. You may not use calculators, notes, or other references. Please show your work and explain your answers; answers with no explanation will not receive full credit. Good luck!

Determine the interval of convergence of the following power series.

\[
\sum_{n=1}^{\infty} \frac{2^n}{9n} (7 - x)^n
\]

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{\frac{2^{n+1}}{9(n+1)} (7 - x)^{n+1}}{\frac{2^n}{9n} (7 - x)^n} \right| = \lim_{n \to \infty} \frac{2}{7-x} \left| \frac{n}{n+1} \right|^{\frac{1}{n}}
\]

\[
\Rightarrow \left| \frac{2}{7-x} \right| < 1
\]

\[
|7-x| < \frac{1}{2} \Rightarrow 6.5 < x < 7.5
\]

Endpoints?

\[
\sum_{n=1}^{\infty} \frac{2^n}{9n} (\frac{7}{3})^n
\]

\[
\sum_{n=1}^{\infty} \frac{1}{9n}
\]

\[
\text{diverges, p-series}
\]

\[
\sum_{n=1}^{\infty} \frac{(-1)^n}{9n}
\]

\[
\text{converges, alternating series test.}
\]

\[
\Rightarrow \text{interval is } 6.5 < x < 7.5
\]

\[
\text{or } x \in [6.5, 7.5]
\]