Math 100 - Week 10 Recitation (Fall 2013)

1. For each of the following alternating series, (a) estimate the sum of the series using the first three terms, and (b) determine an error bound for your estimate.

   (a)
   $$\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n}$$

   (b)
   $$\sum_{n=2}^{\infty} \frac{(-1)^n}{n!}$$

2. Consider the following power series, which converges for all values of x:
   $$\sum_{n=0}^{\infty} \frac{(x-6)^n}{2^n n!}$$

   (a) When we plug in $x = 5$, the resulting series is alternating. How many terms are necessary to estimate the sum of this series and ensure an error less than 0.002 (1/500)?

   (b) When we plug in $x = 5.9$, the resulting series is alternating. How many terms are necessary to estimate the sum of this series and ensure an error less than 0.002 (1/500)?

   (c) Which of these two x-values causes the series to converge more quickly?

3. Consider the following power series:
   $$\sum_{n=0}^{\infty} \frac{(2x+5)^{n+1} n^{10}}{(n+1)!}$$

   (a) At what x-value is this power series centered?

   (b) What is the interval of convergence of this power series?

4. Suppose that the function $f(x)$ has the following Taylor series:
   $$\sum_{n=0}^{\infty} \frac{3^n (x-2)^n}{n^2+3}$$

   (a) Determine the interval of convergence of the series.

   (b) Find the Taylor series for $f'(x)$.

   (c) Determine the interval of convergence for the series you obtained in (b).

5. Let $f(x) = \ln(2x+1)$.

   (a) Find a formula for $f^{(n)}(4)$ when $n > 0$. What is $f^{(n)}(4)$ when $n = 0$?

   (b) Find the Taylor series for $f(x)$ centered at $x = 4$. Use sigma notation, though you may have to include the constant term separately.

   (c) Find the power series which is the (term-by-term) derivative of your series from part (b). What function should this series represent?