Math 100 - Week 8 Recitation (Fall 2013)

For each problem, for parts (a) and (b), determine whether the given series converges or diverges. For part (c), determine which values for the constant C would cause the series to converge. The value of C is not required to be a whole number. The best approach is to solve parts (a) and (b) first, which have different numbers in place of C; think about how changing that value affects convergence, and then which choices of C would cause the series to converge.

To accomplish these tasks, you should not need to use any convergence tests other than the Integral Test and the two Comparison Tests. (And the p-series Test, which is really just a handy application of the Integral Test.)

1. (a) $$\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^5+6n^2}}$$ (b) $$\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{2n^5+6n^2}}$$ (c) $$\sum_{n=1}^{\infty} \frac{n^C}{\sqrt{2n^5+6n^2}}$$

2. (a) $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/3}}$$ (b) $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{7/3}}$$ (c) $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{C}}$$

3. For this problem, assume $f(x)$ is a continuous function such that for all x,
$$2x^4 \le f(x) \le 4x^4.$$
(a) $$\sum_{n=1}^{\infty} \frac{f(n)}{n^2}$$ (b) $$\sum_{n=1}^{\infty} \frac{f(n)}{n^{10}}$$ (c) $$\sum_{n=1}^{\infty} \frac{f(n)}{n^C}$$

4. (a) $$\sum_{n=1}^{\infty} \sin(1/n)$$ (b) $$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^4}$$ (c) $$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^C}$$

(Hint for Problem 4: For part (a), use the Limit Comparison Test to compare the series to $\sum(1/n)$*. Use this as a guide when choosing comparisons for parts (b) and (c).)*