If, when solving these problems, you encounter an integral you’ve already evaluated in an earlier problem on the worksheet, you should use that result, rather than solving the same integral a second time. Knowing (or referencing a list of) trigonometric identities will be useful frequently.

1. Evaluate the integral
\[ \int \sin^2 \theta \, d\theta \]
in two different ways:
   (a) Apply a trigonometric identity, then integrate.
   (b) Integrate by parts, then apply the identity \( \sin^2 x + \cos^2 x = 1 \) to the result, and solve for the value of the original integral.

Are your answers the same (or at least mathematically equivalent)?

2. Evaluate the integral
\[ \int \tan \theta \sec^2 \theta \, d\theta \]
in two different ways, using two different \( u \)-substitutions. Are your answers the same (or at least mathematically equivalent)?

3. Evaluate the following integral:
\[ \int \frac{x^2}{\sqrt{1-x^2}} \, dx \]

4. Evaluate the following integral:
\[ \int x \sin^{-1} x \, dx \]

5. An ellipse has an equation of the form:
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Find the area of the ellipse as follows:
   (a) Solve for \( y \) in terms of \( x \) (you should get different solutions for the top and bottom halves).
   (b) Set up a definite integral that represents either one-half or one-quarter of the total area.
   (c) Evaluate the integral and find the area.

6. Devise two different strategies that would allow you to evaluate the following integral:
\[ \int \frac{\sin^{13} x}{\cos^{15} x} \, dx \]

Is one approach easier to execute than the other?