Math 100 - Week 4 Recitation (Fall 2013)

If, when solving these problems, you encounter an integral you've already evaluated in an earlier problem on the worksheet, you should use that result, rather than solving the same integral a second time. Knowing (or referencing a list of) trigonometric identities will be useful frequently.

1. Evaluate the integral
$$\int \sin^2\theta\, d\theta$$
in two different ways:
   (a) Apply a trigonometric identity, then integrate.
   (b) Integrate by parts, then apply the identity $\sin^2 x + \cos^2 x = 1$ to the result, and solve for the value of the original integral.

   Are your answers the same (or at least mathematically equivalent)?

2. Evaluate the integral
$$\int \tan\theta \sec^2\theta\, d\theta$$
in two different ways, using two different u-substitutions. Are your answers the same (or at least mathematically equivalent)?

3. Evaluate the following integral:
$$\int \frac{x^2}{\sqrt{1-x^2}}\, dx$$

4. Evaluate the following integral:
$$\int x \sin^{-1} x\, dx$$

5. An ellipse has a equation of the form:
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
Find the area of the ellipse as follows:
   (a) Solve for y in terms of x (you should get different solutions for the top and bottom halves).
   (b) Set up a definite integral that represents either one-half or one-quarter of the total area.
   (c) Evaluate the integral and find the area.

6. Devise two different strategies that would allow you to evaluate the following integral:
$$\int \frac{\sin^{13} x}{\cos^{15} x}\, dx$$
Is one approach easier to execute than the other?