

Friday 3 August 2012

Exam IV

60 minutes

Name:

Instructions:

This is the fourth test of the course. This test focuses on conics and the material from chapter 9. Be calm and think through each question - some may have 'trick' parts. Do not be afraid to skip and come back to different questions or to ask for help/clarifications, and note that it is not necessarily true that questions get harder as the test progresses. I highly recommend showing clear and careful work - partial credit will only come from clear thought processes on paper.

- (1) Do your (own) best work, and draw a box around your final answers.
- (2) Extra paper is available if you need it.
- (3) Show all the steps of your work clearly and label everything so that anyone who knows the material would be able to understand the answer.
- (4) This exam is a calculator-exam – you may use your calculator on every question.
- (5) Ask any questions or raise any concerns whenever you have any doubt.
- (6) **No work = no credit.**

Question	Points	Your Score
Q1	20	
Q2	16	
Q3	18	
Q4	12	
Q5	16	
Q6	18	
TOTAL	100	

[Q1]... [20 points] Conic definitions

[1] What is our definition of a circle? What is the standard form of an equation for a circle?

[2] What is our definition of a parabola? What is the standard form of an equation for a parabola? Give both cases, where the parabola opens vertically or horizontally.

[3] What is our definition of an ellipse? What is the standard form of an equation for an ellipse?

[4] What is our definition of a hyperbola? What is the standard form of an equation for a hyperbola? Give both the cases, where the hyperbola opens vertically or horizontally.

[Q2]... [16 points] Conic Identification I

[1] Write the following equation in standard form, identify what type of conic it is, and identify its foci, directrix, vertices, center, radii, major and minor axes, eccentricity, and asymptotes, if it has any. Finally, on a sheet of graph paper, include a sketch of the conic.

(1)
$$x^2 + y^2 - 4x + 2y - 4 = 0$$

[2] For the conic described by 1 above, find its x and y intercepts, if it has any.

[3] What is the standard form of the equation of the parabola with focus $(8, -2)$ and directrix $x = 4$?

[Q3]... [18 points] Conic Identification II

[1] Write the following equation in standard form, identify what type of conic it is, and identify its foci, directrix, vertices, center, radii, major and minor axes, eccentricity, and asymptotes, if it has any. Finally, on a sheet of graph paper, include a sketch of the conic.

(2)
$$y^2 + 8x + 6y + 25 = 0$$

[2] Find the equation for the tangent line to the conic described by 2 above at the point $(-4, 1)$.

[3] What is the standard form of the equation of the circle with center $(1, -2)$ that goes through the point $(7, 6)$?

[Q4]... [12 points] Conic Identification III

[1] Write the following equation in standard form, identify what type of conic it is, and identify its foci, directrix, vertices, center, radii, major and minor axes, eccentricity, and asymptotes, if it has any. Finally, on a sheet of graph paper, include a sketch of the conic.

(3)
$$9y^2 - x^2 + 2x + 54y + 62 = 0$$

[2] For the conic described by 3 above, find its x and y intercepts, if it has any.

[3] (*extra*) What is your favorite conic, and in exactly 11 syllables describe why.

[Q5]... [16 points] Conic Identification IV

[1] Write the following equation in standard form, identify what type of conic it is, and identify its foci, directrix, vertices, center, radii, major and minor axes, eccentricity, and asymptotes, if it has any. Finally, on a sheet of graph paper, include a sketch of the conic.

(4)
$$16x^2 + 25y^2 - 32x + 50y + 16 = 0$$

[2] For the conic described by 4 above, find its x and y intercepts, if it has any.

[3] What is the standard form of the equation of the ellipse with foci $(1, 1)$ and $(1, -1)$, with major axis of length 8?

[Q6]... [18 points] Parametric and Polar Equations

[1] What type of conic is described by the parametric equation $(t^2, 2t + 1)$?

[2] What type of conic is described by the parametric equation $x = 3 - \cos t, y = 2 + \sin t$?

[3] Perform the following polar-rectangular conversions?

- (1) Convert the rectangular coordinate $(3, 6)$ into polar coordinates.
- (2) Convert the polar coordinate $(3, 2\pi/3)$ into rectangular coordinates.

[4] (*some extra credit here too*) Using any method you know how (including your calculator), provide sketches of the following polar equations:

- (1) $r = 2 \cos 3\theta$
- (2) $r = e^{\cos \theta} - 2 \cos 4\theta + \sin^5(\theta/12)$ - the "butterfly curve"
- (3) $r = 1 + \cos^{99} \theta$ - something I might call Pac-Man, almost.