

1) $\sum_{n=1}^{\infty} \frac{1}{3n^2+1} \sim \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges p-series $\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{3n^2+1}$ converges absolutely
 limit comparison
 as $\frac{n^2}{3n^2+1} \rightarrow \frac{1}{3}$
 (don't need to do any more work).

2) $\sum_{n=1}^{\infty} \frac{1}{35n+1} \sim \sum_{n=1}^{\infty} \frac{1}{5n}$ diverges p-series $\sum_{n=1}^{\infty} \frac{(-1)^n}{35n+1}$ converges by alternating series test.
 limit comparison
 as $\frac{5n}{35n+1} \rightarrow \frac{1}{3}$
 (converges conditionally)

3) $\sum_{n=1}^{\infty} \frac{n!}{3^n(n+3)!}$ Ratio test: $\Rightarrow \left| \frac{(n+1)}{3(n+3)} \right| \rightarrow \frac{1}{3} < 1$ so converges $\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n n!}{3^n(n+3)!}$ converges absolutely.

4) $\sum_{n=1}^{\infty} \frac{(n!)^2}{3^n(n+3)!}$ Ratio test $\Rightarrow \left| \frac{(n+1)^2}{3(n+3)} \right| \rightarrow \infty$ diverges (+ in fact, $\lim a_n \rightarrow \infty$) $\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2}{3^n(n+3)!}$ diverges.

5) $\sum_{n=1}^{\infty} \frac{(2n^2+1)^{3n}}{(3n^3+15)^{2n}}$ Root test $\Rightarrow \lim \left| \frac{(2n^2+1)^3}{(3n^3+15)^2} \right| \sim \frac{8n^6}{9n^6} \rightarrow \frac{8}{9} < 1$ so converges absolutely $\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{(2n^2+1)^{3n}}{(3n^3+15)^{2n}}$ converges absolutely.

6) $\sum_{n=1}^{\infty} \frac{1}{7^n n} < \sum_{n=1}^{\infty} \frac{1}{7^n}$ geometric converges $\sum_{n=1}^{\infty} \frac{(-7)^n}{7^n n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges conditionally (not absolutely, by p-series).

7) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{7^n n}$ Ratio test $\Rightarrow \frac{\sqrt{(n+1)^2+4}}{7^{n+1}(n+1)} \cdot \frac{7^n n}{\sqrt{n^2+4}}$
 $\frac{7^{n/2} \sqrt{(1+\frac{1}{n})^2 + \frac{4}{n^2}}}{7^{n/2} \sqrt{1+\frac{4}{n^2}}} \cdot \frac{n}{n(1+\frac{1}{n})} = \frac{1}{7} = \frac{1}{7} < 1$ converges.

part b diverges by n^{th} term test.

~~at 2:30~~

