Math 100 Week 8 Recitation

7 November 2013

1) (a) $\sum \frac{n}{\sqrt{2n^5+6n^3}} \sim \sum \frac{n}{n^{5/2}} = \sum \frac{1}{n^{3/2}}$ converges by p-series

limit comparison

as $\frac{n \cdot n^{5/2}}{n\sqrt{2n^5+6n^3}} \longrightarrow \frac{1}{\sqrt{2}}$

(b) Similarly, (b) diverges by limit comparison to $\sum \frac{n}{n^{3/2}} = \sum \frac{1}{\sqrt{n}}$.

(c) $\sum_{n=1}^{\infty} \frac{n^c}{\sqrt{2n^5+6n^3}} \sim \sum \frac{n^c}{n^{5/2}}$

limit comparison, as $\frac{\cancel{n^c}\, n^{5/2}}{\sqrt{2n^5+6n^3}\ \cancel{n^c}} \longrightarrow \frac{1}{\sqrt{2}}$ as above.

+ $\sum \frac{n^c}{n^{5/2}} = \sum \frac{1}{n^{5/2-c}}$ which is a p-series, + so converges iff $5/2 - c > 1$, i.e. $5/2 - 1 > c$, i.e.

$\boxed{c < 3/2}$.

2) We'll skip to (c).

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^c}$ integral test: $\int_2^{\infty} \frac{1}{x(\ln x)^c}\,dx = \int_{\ln 2}^{\infty} \frac{1}{u^c}\,du = \left.\frac{u^{-c+1}}{1-c}\right|_{\ln 2}^{\infty}$

$u = \ln x$

$du = \frac{1}{x}$

which converges iff $-c+1<0$, i.e. $\boxed{c>1}$.

3) $2x^4 \le f(x) \le 4x^4$.

$\sum \frac{2n^4}{n^c} \le \sum_{n=1}^{\infty} \frac{f(n)}{n^c} \le \sum \frac{4n^4}{n^c} \Longrightarrow \sum \frac{2}{n^{c-4}} < \sum \frac{f(n)}{n^c} < \sum \frac{4}{n^{c-4}}$

diverges if $c \le 4$ converges if $c > 4$

so it converges iff $c>4$ by 2 basic comparisons.

4) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{n^c} \sim \sum \frac{\frac{1}{n}}{n^c}$

limit comparison, as

L'Hôpital's Rule

$$\lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}} \cdot \frac{\cancel{n^c}}{\cancel{n^c}} = \frac{\cos(\frac{1}{n}) \cancel{\frac{d}{dn}\frac{1}{n}}}{\cancel{\frac{d}{dn}\frac{1}{n}}} \to 1$$

$$+ \lim_{n\to\infty} \cos(\tfrac{1}{n}) = \cos(0) = 1.$$

So $\displaystyle\sum \frac{\sin(\frac{1}{n})}{n^c} \sim \sum \frac{1}{n^{c+1}}$ which converges by p-series iff $\boxed{c>0}$.