Week 7 Recitation 7 November 2013

(a) sequence + limit (b) series + sum

1) $\frac{2^{2n}+3^n}{5^{n-1}} = \frac{2^{2n}}{5^{n-1}} + \frac{3^n}{5^{n-1}} = \frac{4^n}{5^{n-1}} + \frac{3^n}{5^{n-1}} \longrightarrow 0.$

$$\sum_{n=1}^{\infty}\left(\frac{4^n}{5^{n-1}} + \frac{3^n}{5^{n-1}}\right) = \sum_1^{\infty} \frac{4^n}{5^{n-1}} + \sum_1^{\infty} \frac{3^n}{5^{n-1}}$$

geometric, ratio $\frac{4}{5}$, 1st term 4

geometric, ratio $\frac{3}{5}$, 1st term 3

$\Longrightarrow$ sum is $\boxed{\frac{4}{1-\frac{4}{5}} + \frac{3}{1-\frac{3}{5}}}$ (converges, as both ratios < 1).

2) $\frac{2^n+3^{2n}}{5^{n+1}} = \underbrace{\frac{2^n}{5^{n+1}}}_{\to 0} + \frac{9^n}{5^{n+1}} \longrightarrow \infty.$ so sequence diverges.

By n^{th} term test, $\sum \frac{2^n+3^{2n}}{5^{n+1}}$ diverges. //

3) $\lim_{n\to\infty} \frac{4e^n+n^2}{9e^n+2n} \overset{\text{L'Hôpital}}{=} \frac{4e^n+2n}{9e^n+2} \overset{\text{L'Hôpital}}{=} \frac{4e^n+2}{9e^n} = \frac{4e^n}{9e^n} + \underbrace{\frac{2}{9e^n}}_{\to 0} = \frac{4}{9}$

By n^{th} term test, the series diverges.

4) $\frac{1}{(n^3+1)^{1/3}} - \frac{1}{((n+2)^3+1)^{1/3}} \longrightarrow 0.$

The series telescopes, + starts $\left(\frac{1}{2^{1/3}} - \cancel{\frac{1}{(3^3+1)^{1/3}}}\right) + \frac{1}{(2^3+1)^{1/3}} - \cancel{\frac{1}{(4^3+1)^{1/3}}} + \cancel{\frac{1}{(3^3+1)^{1/3}}} - \cancel{\frac{1}{(5^3+1)^{1/3}}}$

so sum is $\frac{1}{2^{1/3}} + \frac{1}{(2^3+1)^{1/3}}$ //

5. The sequence here is totally unreasonable. But you can do them with Taylor series.

$$(n^3+1)^{1/3} = n + \frac{1}{3n^2} + -\frac{1}{9n^5} + \cdots$$

$$((n+2)^3+1)^{1/3} = n+2 + \frac{1}{3n^2} - \frac{4}{3n^3} + \frac{4}{x^4} + \cdots$$

being very fancy w/ Taylor's Theorem (or using what your book calls the generalized binomial theorem).

$$\Rightarrow \lim_{n\to\infty} (n^3+1)^{1/3} - ((n+2)^3+1)^{1/3} = \left(\cancel{n} + \cancel{\tfrac{1}{3n^2}}^{\,0} + \cancel{\cdots}^{\,0}\right) - \left(\cancel{n} + \underline{2} + \cancel{\tfrac{1}{3n^2}}^{\,0} + \cancel{\cdots}^{\,0}\right)$$

$$= -2.$$

So the sequence $\longrightarrow -2$. But I don't expect you to do that. //

The series telescopes, but in the wrong (diverging) way.

(+ it fails n^{th} term test) $\longrightarrow$ diverges. //

6. $\frac{2^n+1}{e^n} = \left(\frac{2}{e}\right)^n + \left(\frac{1}{e}\right)^n \longrightarrow 0$

geometric series $\Rightarrow$ sum is $\frac{2/e}{1-2/e} + \frac{1/e}{1-1/e}$ ■

7. $\frac{4^n+1}{\pi^n} = \left(\frac{4}{\pi}\right)^n + \left(\frac{1}{\pi}\right)^n$ $\quad 4 > \pi$, so $\left(\frac{4}{\pi}\right)^n \to \infty$

so sequence diverges & thus the series diverges.

8. $\ln(n(n+1)) = \ln n + \ln n+1$
$\ln((n+1)^2) = 2\ln(n+1)$

$\Rightarrow \ln(n(n+1)) - \ln((n+1)^2) = \ln n - \ln(n+1)$
$= \ln\left(\frac{n}{n-1}\right) \longrightarrow 0.$

the series is $\sum \ln n - \ln(n+1)$, which telescopes, but in a divergent way.

So it diverges.