Brown University - Math 0090

FINAL EXAM - December 21, 2012

NAME:

SECTION (Number and Professor):

Do not open the exam until you are told to do so. Please write your name and read the instructions on this page until you are instructed to begin.

You will have three hours to complete the exam. There are eleven problems, some of which have multiple parts. Use your time wisely; if you find yourself stuck on any problems, make sure you attempt all of the other problems before you run out of time.

You must show your work. Correct answers with no explanation may not be given any credit, while incorrect or incomplete answers where some (correct) work is shown are likely to receive some partial credit.

You may not refer to external sources such as notes, your textbook or a calculator. If you wish, you may use the backs of the pages for scratchwork. Good luck!

1	2	3	4	5	6	7	8	9	10	11	TOTAL/110

1. Find the derivative of each of the following functions of x:

(a)

$$e^{\arcsin x}$$

(b)

$$\frac{\sin^2 x}{x}$$

(c)

$$\int_0^{x^2} \cos(t^3)\, dt$$

2. Evaluate each of the following integrals:

(a)

$$\int e^{5x}\sqrt{e^{5x}+1}\,dx$$

(b)

$$\int_1^3 \left(\frac{x^4+x^2+1}{x^2}\right)\,dx$$

(c)

$$\int_1^e \frac{\ln x}{x}\,dx$$

3. A small gang of criminals rob the Thirteenth National Bank in Sparseville, and at midnight they begin driving north at 80 miles per hour. At 1:00am, a police car leaves the police station, which is located 70 miles east of the bank, driving west toward the bank at 40 miles per hour.

   (a) At 1:30am, what is the distance between the criminals' car and the police car? (This should not require any calculus.)

   (b) At 1:30am, at what rate is the distance between the criminals' car and the police car changing? (Be sure to specify whether it is increasing or decreasing.)

4. A particle moving along the x-axis has acceleration

$$a(t) = -6t + 4$$

units per second squared at time t.

At time $t = 0$, the position of the particle is $x = 2$, and at time $t = 1$ the position is $x = 7$.

(a) Find a formula for the position of the particle at time t.

(b) At what time t is the forward velocity of the particle at a maximum?

5. Consider the region enclosed by the vertical line $x = 3$ and the parabola $x = y^2 - 4y + 3$.

   (a) Determine the area of this region.

   (b) Determine the volume of the solid generated by revolving this region around the axis $x = 3$.

6. Use logarithmic differentiation to find the derivative of the following function:

$$f(x) = x^{4x^2}$$

7. For each of the following limits, evaluate the limit or show it does not exist:

(a)

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

(b)

$$\lim_{x \to \infty} \frac{(1 + 2x^2)(3 + 4x)}{5x^3 + 6}$$

(c)

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

8. At a Brown-Harvard basketball game, the Brown Band discovers that the amount of time they play the Brown Fight Song controls the result of the game. If they play the Fight Song for x minutes, the Brown team will score

$$B(x) = -0.5x^2 + 7x + 63.5$$

points, and the Harvard team will score

$$H(x) = -1.5x^2 + 10x + 55$$

points. (It is possible for these functions to result in fractional scores... don't worry about this.) Assume that the band can play the Fight Song for at most ten minutes, primarily due to the very demanding tuba part.

(a) If the band wants Brown to score as many points as possible, how long should they play the Fight Song?

(b) If the band wants Brown to win by as many points as possible, how long should they play the Fight Song?

9. Consider the region enclosed by the x-axis, the vertical lines $x = 1$ and $x = 3$, and the curve $y = \sqrt{x^2 + 8x}$.

   Find the volume of the solid generated by revolving this region around the axis $x = -4$.

10. Let

$$f(x) = \frac{1}{3x+1}$$

(a) Find the derivative $f'(x)$ using the limit definition of the derivative.

(b) Find the derivative $f'(x)$ using the Chain Rule.

11. Let

$$h(x) = \cos x + \frac{x}{2}.$$

(a) Find all critical points of $h(x)$ on the interval $[0, 2\pi]$, and for each one determine whether it is a local minimum, a local maximum, or neither.

(b) Find the absolute maximum and absolute minimum values of $h(x)$ on the interval $[0, \pi/2]$.
(Your results from part (a) should be helpful!)