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1. Let $f(x) = 2x^3 + 3x^2$.

   (a) On which intervals is $f(x)$ increasing or decreasing?

   (b) On which intervals is $f(x)$ concave up or concave down?

   (c) Draw a simple sketch of the curve $y = f(x)$. Label any local maxima, local minima, and/or inflection points (and specify which of these the points are).

2. Evaluate each of the following limits.

   (a)
   $$\lim_{x \to 1} \frac{x \ln x}{\sin(\pi x)}$$

   (b)
   $$\lim_{x \to 0} \frac{\tan^{-1}(2x)}{x^3 + 3x}$$

3. If the Write On! Chalk company buys x kilograms of chalk powder in a month, they can produce $(100x - x^2)$ cartons of chalk, each of which will be sold for \$5. Suppose that chalk powder costs \$30 per kilogtam, and that it also costs the company \$1000 to run their factory for the month (no matter how much chalk they make).

   (a) Determine a function $P(x)$ that represents the profit the company makes if they buy x kilograms of chalk powder.
   (Hint: Profit is [money taken in] minus [money spent].)

   (b) Determine the number of kilograms of powder the company should buy in order to maximize their profit.

4. Suppose that $f(x)$ is a continuous function such that $f''(x) = x^3 + e^x + 1$ and $f'(2) = 0$.

   (a) Determine the function $f'(x)$.

   (b) Find two different possible functions that could be $f(x)$ in this case.

   (c) Based on this information, $f(x)$ has exactly one local extreme. At which value of x does it occur, and is it a local maximum or a local minimum?

5. Let $h(x) = 4x + 2\ln x - 5$.

   (a) Show that $h(x) = 0$ for at least one x in the interval $[1, 2]$.

   (b) Show that $h(x) = 0$ for *exactly* one x in the interval $[1, 2]$.
   (Hint: Find $f'(x)$. What is true about the derivative for values of x in this interval?)

6. Let $f(x) = e^{-2x}(x^2 - 4x + 2)$.

   (a) Find all critical points of $f(x)$.

   (b) Determine the absolute minimum and maximum values of $f(x)$ on the interval $[0, 3]$.

7. A water filtration system consists of an inverted conical tank (pointed end down) suspended over a cylindrical tank; water drains out of the bottom of the cone and falls into the cylinder below. The cone has a base radius of 20 meters and a height of 10 meters; the cylinder has a radius of 40 meters.

   At the moment the water level in the cone is at a height of 5 meters, the water level is dropping at a rate of 6 meters per second. Let U be the volume of water in the cone, and let L be the volume of water in the cylinder.

   (a) At the moment described above, how fast is the volume of water in the cone changing?

   (b) At all times, how are dU/dt and dL/dt related? (Your answer should be in the form of an equation.)

   (c) At the moment described above, how fast is the height of the water in the cylinder changing?